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Optimization of beamforming and path planning for UAV-assisted wireless relay networks

Ouyang Jian [a], Zhuang Yi [a,*], Lin Min [b,c], Liu Jia [d]

[a] *College of Computer Science and Technology, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China*
[b] *PLA University of Science and Technology, Nanjing 210007, China*
[c] *National Mobile Communications Research Laboratory, Southeast University, Nanjing 210096, China*
[d] *Computer Center, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China*

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Abstract Recently, unmanned aerial vehicles (UAVs) acting as relay platforms have attracted considerable attention due to the advantages of extending coverage and improving connectivity for long-range communications. Specifically, in the scenario where the access point (AP) is mobile, a UAV needs to find an efficient path to guarantee the connectivity of the relay link. Motivated by this fact, this paper proposes an optimal design for beamforming (BF) and UAV path planning. First of all, we study a dual-hop amplify-and-forward (AF) wireless relay network, in which a UAV is used as relay between a mobile AP and a fixed base station (BS). In the network, both of the AP and the BS are equipped with multiple antennas, whereas the UAV has a single antenna. Then, we obtain the output signal-to-noise ratio (SNR) of the dual-hop relay network. Based on the criterion of maximizing the output SNR, we develop an optimal design to obtain the solution of the optimal BF weight vector and the UAV heading angle. Next, we derive the closed-form outage probability (OP) expression to investigate the performance of the dual-hop relay network conveniently. Finally, computer simulations show that the proposed approach can obtain nearly optimal flying path and OP performance, indicating the effectiveness of the proposed algorithm. Furthermore, we find that increasing the antenna number at the BS or the maximal heading angle can significantly improve the performance of the considered relay network.

1. Introduction

It is well-known that unmanned aerial vehicles (UAVs) have attracted considerable attention in both military and civilian applications, such as surveillance and reconnaissance,[1] emergency rescue,[2] and data collection and transmission,[3] since the UAV is regarded as a suitable "airborne communication relay" to provide reliable connectivity of the link between a base station (BS) and an access point (AP) which are separated by a distance beyond the range of communication devices or

* Corresponding author. Tel.: +86 25 84896779.
E-mail address: zhuangyi@nuaa.edu.cn (Y. Zhuang).

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shadowed by urban or mountainous terrain (e.g., forests, buildings). Under this situation, there are a lot of works considering how to improve the performance of UAV-assisted wireless networks recently. For example, Zhu et al.[4] have studied some types of connectivity in mobile ad hoc networks, and have presented several approaches based on the graph theory to optimize the movement of a UAV to improve the connectivity. de Freitas et al.[5] have considered the scheme of applying multiple UAVs as mobile relays to maintain the connectivity of links between isolated APs and a BS. For the case of using one or more intermediate UAVs as a relay chain, many schemes have been presented to optimize the network performance. Burdakov et al.[6] have presented label correcting algorithms to find a Pareto solution to the two-objective problem of minimizing the path cost and the number of UAVs. Based on graph search, Burdakov et al.[7] have proposed an algorithm for generating Pareto-optimal relay chains to achieve a good tradeoff between the number of UAVs and the quality of the chain. By estimating the communication objective function gradient, a decentralized mobility control algorithm has been presented for formatting and maintaining an optimal communication chain with a team of UAVs.[8] Du and Zhang[9] have investigated the network-coded protocol of aeronautical communications, and presented a selection algorithm based on the outage probability (OP) for a relay network. However, the aforementioned works assume that the ground APs are static. More recently, there are increasing interests in the scenario where the APs (vehicles, aircrafts, etc.) are mobile. Due to the mobility of the APs, the communication environment and network topology constantly change and thus the UAVs need to adjust their paths in time to best accommodate the evolving network environment. By using a multi-antenna UAV as the relay between ground APs and a BS, a heading optimization approach has been proposed to maximize the sum rate of the AP-UAV link.[10] In addition, the performance enhancement through orthogonal space–time block coding (OSTBC) transmission has also been examined. Jiang and Swindlehurst[11] have considered a relay system with a multi-antenna UAV and some single-antenna ground APs, and developed an algorithm to maximize the approximate ergodic sum rate of the AP-UAV link by adjusting the UAV heading.

Although the previous works have significantly improved our understanding on the schemes and performances of UAV-assisted wireless networks,[10,11] their main drawback is that they have only investigated the AP-UAV single-hop link transmission. Actually, wireless networks using a UAV as a relay should be modeled as a dual-hop relaying.[12,13] Inspired by these observations, we study a dual-hop relay network, in which both of the AP and the BS are equipped with multiple antennas to obtain the array gain, while the UAV with a single antenna applies an amplify-and-forward (AF) protocol to assist the signal transmission. Specifically, we firstly derive the output signal-to-noise ratio (SNR) of the considered relay network. Then, by using the criterion of maximizing the output SNR, an optimal design for beamforming (BF) and UAV path planning is presented. Next, we derive the closed-form expression of the OP to evaluate the dual-hop relay system performance. It is worth mentioning that in Refs.[12,13] all of the nodes are assumed to have a single antenna, and the optimization of the UAV flying path has not been studied. Besides, the performance of the relay network has only been examined through Monte Carlo simulations in Ref.[12], and the decode-and-forward (DF) protocol has been adopted for the UAV in Ref.[13].

2. Problem formulation

As shown in Fig. 1, we consider a wireless network in which a mobile AP communicates with a fixed BS through a UAV acting as a relay. We assume that the AP and the BS are equipped with N_1 and N_2 antennas, respectively, and the UAV has a single antenna due to the payload limitation. The communication from the AP to the BS takes place in two time slots. During the first time slot, the AP performs transmit BF and sends its signals to the UAV. Accordingly, the received signals at the UAV can be written as

$$y_r(t) = \sqrt{P_1}\boldsymbol{h}_1^{\mathrm{H}}(t)\boldsymbol{w}_1 s(t) + n_1(t) \tag{1}$$

where P_1 denotes the transmit power at the AP, $s(t)$ the transmitted signal with $E[|s(t)|^2] = 1$, $\boldsymbol{w}_1$ the $N_1 \times 1$ normalized transmit BF weight vector obeying $||\boldsymbol{w}_1||_F^2 = 1$, and $n_1(t)$ the white Gaussian noise (AWGN) with zero mean and variance σ_1^2. Through the paper, $(\bullet)^{\mathrm{H}}$ stands for the Hermitian transpose, $E[\bullet]$ the expectation, $|\bullet|$ the absolute value, and $||\bullet||_F$ the Frobenius norm of a matrix. Furthermore, $\boldsymbol{h}_1(\mathrm{t})$ $N_1 \times 1$ is the channel vector of the AP-UAV link with path loss, which is given by[14]

$$\boldsymbol{h}_1(t) = \frac{\boldsymbol{g}_1(t)}{d_1^{\alpha}} \tag{2}$$

where $\boldsymbol{g}_1(t)$ represents the $N_1 \times 1$ channel vector with Rayleigh fading entries, d_1 the distance between the AP and the UAV, and α the path loss exponent. During the second time slot, the UAV amplifies the received signals by a gain factor as

$$G = \sqrt{\frac{1}{P_1|\boldsymbol{h}_1^{\mathrm{H}}(t)\boldsymbol{w}_1|^2}} \tag{3}$$

and forwards them to the BS. By applying BF, the received signals at the BS can be written as

$$\begin{aligned} y_d(t) &= \boldsymbol{w}_2^{\mathrm{H}}(\sqrt{P_2}G\boldsymbol{h}_2(t)y_r(t) + \boldsymbol{n}_2(t)) \\ &= \sqrt{P_1P_2}G\boldsymbol{w}_2^{\mathrm{H}}\boldsymbol{h}_2(t)\boldsymbol{h}_1^{\mathrm{H}}(t)\boldsymbol{w}_1 s(t) + \sqrt{P_2}G\boldsymbol{w}_2^{\mathrm{H}}\boldsymbol{h}_2(t)n_1(t) \\ &\quad + \boldsymbol{w}_2^{\mathrm{H}}\boldsymbol{n}_2(t) \end{aligned} \tag{4}$$

where P_2 represents the transmit power at the UAV, $\boldsymbol{w}_2$ the $N_2 \times 1$ normalized receive BF weight vector satisfying $||\boldsymbol{w}_2||_F^2 = 1$, $\boldsymbol{n}_2(t)$ the $N_2 \times 1$ AWGN vector satisfying $E[\boldsymbol{n}_2(t)\boldsymbol{n}_2^{\mathrm{H}}(t)] =$

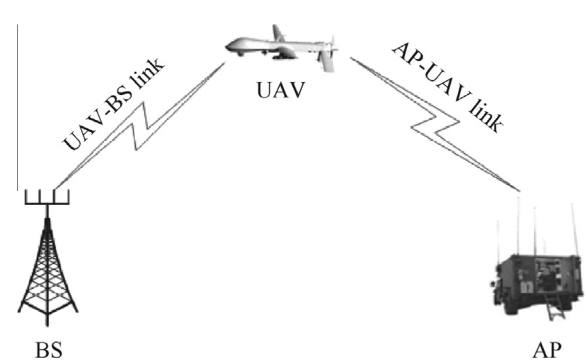


Fig. 1 Application of a UAV as a relay in a wireless network.

$\sigma_2^2\boldsymbol{I}_{N_2}$ with $\boldsymbol{I}_{N_2}$ the $N_2\times N_2$ identity matrix, and $\boldsymbol{h}_2(t)$ the $N_2\times 1$ channel vector of the UAV-BS link described by[14]

$$\boldsymbol{h}_2(t)=\frac{\boldsymbol{g}_2(t)}{d_2^{\alpha}} \tag{5}$$

where d_2 is the distance between the UAV and the BS, and $\boldsymbol{g}_2(t)$ the $N_2\times 1$ channel vector with Rayleigh fading entries. The index of t will be dropped from now on for notational brevity.

By performing some algebraic manipulations, the output SNR of the dual-hop AF relay network can thus be mathematically expressed as

$$\begin{aligned}\gamma&=\frac{P_1P_2\boldsymbol{w}_2^{\mathrm{H}}\boldsymbol{h}_2\boldsymbol{h}_1^{\mathrm{H}}\boldsymbol{w}_1\boldsymbol{w}_1^{\mathrm{H}}\boldsymbol{h}_1\boldsymbol{h}_2^{\mathrm{H}}\boldsymbol{w}_2}{P_2\sigma_1^2\boldsymbol{w}_2^{\mathrm{H}}\boldsymbol{h}_2\boldsymbol{h}_2^{\mathrm{H}}\boldsymbol{w}_2+\sigma_2^2/G^2}\\&=\frac{\bar{\gamma}_1\bar{\gamma}_2\boldsymbol{w}_2^{\mathrm{H}}\boldsymbol{g}_2\boldsymbol{g}_1^{\mathrm{H}}\boldsymbol{w}_1\boldsymbol{w}_1^{\mathrm{H}}\boldsymbol{g}_1\boldsymbol{g}_2^{\mathrm{H}}\boldsymbol{w}_2}{\bar{\gamma}_1d_2^{2\alpha}\boldsymbol{g}_1^{H}\boldsymbol{w}_1\boldsymbol{w}_1^{\mathrm{H}}\boldsymbol{g}_1+\bar{\gamma}_2d_1^{2\alpha}\boldsymbol{w}_2^{\mathrm{H}}\boldsymbol{g}_2\boldsymbol{g}_2^{\mathrm{H}}\boldsymbol{w}_2}\end{aligned} \tag{6}$$

where $\bar{\gamma}_i=P_i/\sigma_i^2(i=1,2)$ is the average SNR per hop. Compared with the output SNR expressions in Refs.[10,11], it is obvious that our work extends the single-hop link to a more general and actual dual-hop link for a UAV-assisted relay network in this paper. In addition, it should be pointed out that our work can be easily extended to the scenario of employing multiple APs, which has to be ignored here for the purpose of simplicity and clear discussion.

3. Optimization of the UAV-assisted relay network

In this section, based on the criterion of maximizing the output SNR in Eq. (6), we present an optimal algorithm for the UAV-assisted dual-hop AF relay network, by determining the BF weight vectors, $\boldsymbol{w}_{1,T}$ and $\boldsymbol{w}_{2,T}$, and the flying path of the UAV with respect to time step T.

According to Eq. (6), the optimization problem to obtain the maximal output SNR at time step T can be expressed as

$$\gamma_T=\max_{\boldsymbol{w}_{i,T},d_{i,T}}\frac{\bar{\gamma}_1\bar{\gamma}_2\boldsymbol{w}_{2,T}^{\mathrm{H}}\boldsymbol{g}_{2,T}\boldsymbol{g}_{1,T}^{\mathrm{H}}\boldsymbol{w}_{1,T}\boldsymbol{w}_{1,T}^{\mathrm{H}}\boldsymbol{g}_{1,T}\boldsymbol{g}_{2,T}^{\mathrm{H}}\boldsymbol{w}_{2,T}}{\bar{\gamma}_1d_{2,T}^{2\alpha}\boldsymbol{g}_{1,T}^{\mathrm{H}}\boldsymbol{w}_{1,T}\boldsymbol{w}_{1,T}^{\mathrm{H}}\boldsymbol{g}_{1,T}+\bar{\gamma}_2d_{1,T}^{2\alpha}\boldsymbol{w}_{2,T}^{\mathrm{H}}\boldsymbol{g}_{2,T}\boldsymbol{g}_{2,T}^{\mathrm{H}}\boldsymbol{w}_{2,T}} \tag{7a}$$

$$\text{s.t.}\quad \boldsymbol{w}_{1,T}^{\mathrm{H}}\boldsymbol{w}_{1,T}=\boldsymbol{w}_{2,T}^{\mathrm{H}}\boldsymbol{w}_{2,T}=1 \tag{7b}$$

Obviously, the objective function in Eq. (7a) can be rewritten as

$$\begin{aligned}\max_{\boldsymbol{w}_{i,T},d_{i,T}}\gamma_T&=\min_{\boldsymbol{w}_{i,T},d_{i,T}}\frac{1}{\gamma_T}\\&=\min_{\boldsymbol{w}_{i,T},d_{i,T}}\frac{d_{1,T}^{2\alpha}}{\bar{\gamma}_1\boldsymbol{w}_{1,T}^{\mathrm{H}}\boldsymbol{g}_{1,T}\boldsymbol{g}_{1,T}^{\mathrm{H}}\boldsymbol{w}_{1,T}}+\frac{d_{2,T}^{2\alpha}}{\bar{\gamma}_2\boldsymbol{w}_{2,T}^{\mathrm{H}}\boldsymbol{g}_{2,T}\boldsymbol{g}_{2,T}^{\mathrm{H}}\boldsymbol{w}_{2,T}}\end{aligned} \tag{8}$$

Since $\boldsymbol{w}_{i,T}$ and $d_{i,T}$ (i = 1, 2) are uncorrelated with each other, we can obtain their optimal solutions independently. We now focus on optimal BF weight vectors $\boldsymbol{w}_{i,T}$ with fixed $d_{i,T}$. By employing Eqs. (8) and (7b), the constrained optimization problem to calculate the BF weigh vectors can be, respectively, expressed as

$$\max_{\boldsymbol{w}_{1,T}}\boldsymbol{w}_{1,T}^{\mathrm{H}}\boldsymbol{g}_{1,T}\boldsymbol{g}_{1,T}^{\mathrm{H}}\boldsymbol{w}_{1,T}\quad \text{s.t.}\quad \boldsymbol{w}_{1,T}^{\mathrm{H}}\boldsymbol{w}_{1,T}=1 \tag{9a}$$

and

$$\max_{\boldsymbol{w}_{2,T}}\boldsymbol{w}_{2,T}^{\mathrm{H}}\boldsymbol{g}_{2,T}\boldsymbol{g}_{2,T}^{\mathrm{H}}\boldsymbol{w}_{2,T}\quad \text{s.t.}\quad \boldsymbol{w}_{2,T}^{\mathrm{H}}\boldsymbol{w}_{2,T}=1 \tag{9b}$$

As for Eq. (9a), applying eigen-decomposition to the rank-one matrix $\boldsymbol{g}_{1,T}\boldsymbol{g}_{1,T}^{\mathrm{H}}$ yields

$$\begin{cases}\boldsymbol{g}_{1,T}\boldsymbol{g}_{1,T}^{\mathrm{H}}=\boldsymbol{V}_1\boldsymbol{\Sigma}_1\boldsymbol{V}_1^{\mathrm{H}}\\ \boldsymbol{\Sigma}_1=\mathrm{diag}(||\boldsymbol{g}_{1,T}||_F^2,0,\ldots,0)\end{cases} \tag{10}$$

where $\boldsymbol{V}_1=[\boldsymbol{v}_{1,1}\boldsymbol{v}_{1,2}\ldots\boldsymbol{v}_{1,N_1}]$ is the unitary matrix with $\boldsymbol{v}_{1,1}=\boldsymbol{g}_{1,T}/||\boldsymbol{g}_{1,T}||_F^2$ and diag(a_1, a_2, ..., a_N) is an $N\times N$ sized diagonal matrix with a_1, a_2, ..., a_N being its diagonal elements. Then, we introduce the following proposition, which has been proven in Ref.[15]

Proposition 1. *Suppose $\boldsymbol{H}$ is an $N\times N$ Hermitian matrix, the following inequality holds*

$$\frac{\boldsymbol{x}^{\mathrm{H}}\boldsymbol{Hx}}{\boldsymbol{x}^{\mathrm{H}}\boldsymbol{x}}\leqslant\lambda_{\max}(\boldsymbol{H}) \tag{11}$$

which is satisfied with equality at the optimum only when

$$\boldsymbol{x}^{\mathrm{opt}}=\boldsymbol{u}_{\max}(\boldsymbol{H}) \tag{12}$$

where $\lambda_{\max}(\boldsymbol{H})$ and $\boldsymbol{u}_{\max}(\boldsymbol{H})$ denote the largest eigenvalue and the corresponding eigenvector of the matrix $\boldsymbol{H}$, respectively.

Letting $\boldsymbol{H}=\boldsymbol{g}_{1,T}\boldsymbol{g}_{1,T}^{\mathrm{H}}$ in Eq. (11) and using Eq. (9a) along with Eq. (10), one can easily obtain

$$\frac{\boldsymbol{w}_{1,T}^{\mathrm{H}}\boldsymbol{g}_{1,T}\boldsymbol{g}_{1,T}^{\mathrm{H}}\boldsymbol{w}_{1,T}}{\boldsymbol{w}_{1,T}^{\mathrm{H}}\boldsymbol{w}_{1,T}}\leqslant\lambda_{\max}(\boldsymbol{g}_{1,T}\boldsymbol{g}_{1,T}^{\mathrm{H}})=||\boldsymbol{g}_{1,T}||_F^2 \tag{13}$$

The equality holds only when

$$\boldsymbol{w}_{1,T}^{\mathrm{opt}}=\boldsymbol{u}_{\max}(\boldsymbol{g}_{1,T}\boldsymbol{g}_{1,T}^{\mathrm{H}})=\boldsymbol{g}_{1,T}/||\boldsymbol{g}_{1,T}||_F^2 \tag{14}$$

Similarly, the solution to Eq. (9b) is given by

$$\boldsymbol{w}_{2,T}^{\mathrm{opt}}=\boldsymbol{g}_{2,T}/||\boldsymbol{g}_{2,T}||_F^2 \tag{15}$$

By substituting Eqs. 14 and 15 into Eq. (7a), the maximal output SNR with optimal BF weights at time step T can be written as

$$\gamma_T^{\max}=\frac{\bar{\gamma}_1||\boldsymbol{g}_{1,T}||_F^2\bar{\gamma}_2||\boldsymbol{g}_{2,T}||_F^2}{\bar{\gamma}_1||\boldsymbol{g}_{1,T}||_F^2d_{2,T}^{2\alpha}+\bar{\gamma}_2||\boldsymbol{g}_{2,T}||_F^2d_{1,T}^{2\alpha}} \tag{16}$$

Next, we will solve the optimization of $d_{i,T}^{2\alpha}(i=1,2)$ to obtain the optimal flying path at time step T. Defining the three-dimensional coordinates of the AP, the BS, and the UAV as ($x_{1,T}$, $y_{1,T}$, 0), ($x_{2,T}$, $y_{2,T}$, 0), and (x_T, y_T, h_T), one can obtain

$$d_{i,T}=\sqrt{(x_T-x_{i,T})^2+(y_T-y_{i,T})^2+h_T^2},(i=1,2) \tag{17}$$

By assuming that the UAV flies at a constant altitude h with velocity v and maximal heading angle $\delta_{\max}$, the position of the UAV at time step T with the Dubin's car model[16] can be expressed as

$$\begin{cases}x_T=x_{T-1}+v\Delta T\cos\delta_T\\ y_T=y_{T-1}+v\Delta T\sin\delta_T\end{cases} \tag{18}$$

where $\delta_T\in[\delta_{T-1}-\delta_{\max},\ \delta_{T-1}+\delta_{\max}]$ is the UAV heading angle at time step T and ΔT is the time interval indicating the UAV heading update rate. By substituting Eq. (18) into Eq. (17), it is easy to obtain[11]

$$\begin{aligned}d_{i,T}^{2\alpha}&=\left[(x_{T-1}+v\Delta T\cos\delta_T-x_{i,T})^2+(y_{T-1}+v\Delta T\sin\delta_T-y_{i,T})^2+h^2\right]^{\alpha}\\&=(a_{i,T}+b_{i,T}\cos\delta_T+c_{i,T}\sin\delta_T)^{\alpha}\\&\approx a_{i,T}^{\alpha}+\alpha a_{i,T}^{\alpha-1}(b_{i,T}\cos\delta_T+c_{i,T}\sin\delta_T)\\&=a_{i,T}^{\alpha}+\rho_{i,T}\cos(\delta_T-\phi_{i,T})\end{aligned} \tag{19}$$

where

$$\begin{cases} a_{i,T} = (x_{T-1} - x_{i,T})^2 + (y_{T-1} - y_{i,T})^2 + v^2\Delta T^2 + h^2 \\ b_{i,T} = 2v(x_{T-1} - x_{i,T})\Delta T \\ c_{i,T} = 2v(y_{T-1} - y_{i,T})\Delta T \\ \rho_{i,T} = \alpha a_{i,T}^{\alpha-1}\sqrt{b_{i,T}^2 + c_{i,T}^2} \\ \phi_{i,T} = \begin{cases} \arctan\dfrac{c_{i,T}}{b_{i,T}} & b_{i,T} \geqslant 0 \\ \arctan\dfrac{c_{i,T}}{b_{i,T}} + \pi & b_{i,T} < 0 \end{cases} \end{cases} \tag{20}$$

In deriving Eq. (19), we have applied the Maclaurin expansion. Clearly, $d_{i,T}^{2\alpha}$ is a function of the UAV heading angle δ_T. Therefore, in what follows, based on the criterion of maximizing the output SNR, we will present a path planning method by calculating the optimal value of δ_T .

According to Refs.[10,11], the average output SNR rather than instantaneous SNR should be applied to determine δ_T. To this end, we consider Eq. (16) and express the average maximal SNR at time step T as

$$\bar{\gamma}_T^{\max} = E_{g_{1,T},g_{2,T}}\left(\frac{\bar{\gamma}_1||\boldsymbol{g}_{1,T}||_F^2\bar{\gamma}_2||\boldsymbol{g}_{2,T}||_F^2}{\bar{\gamma}_1||\boldsymbol{g}_{1,T}||_F^2 d_{2,T}^{2\alpha} + \bar{\gamma}_2||\boldsymbol{g}_{2,T}||_F^2 d_{1,T}^{2\alpha}}\right) \tag{21}$$

Due to the very complex calculations of the expectation in Eq. (21), it is not possible to obtain the value of $\bar{\gamma}_T^{\max}$. However, by using the Taylor expansion and taking the first term of the expectation in Eq. (21), the approximate average SNR can be obtained as[17]

$$\begin{aligned} \bar{\gamma}_T^{\max} &\approx \frac{E_{g_{1,T},g_{2,T}}\left(\bar{\gamma}_1||\boldsymbol{g}_{1,T}||_F^2\bar{\gamma}_2||\boldsymbol{g}_{2,T}||_F^2\right)}{E_{g_{1,T},g_{2,T}}\left(\bar{\gamma}_1||\boldsymbol{g}_{1,T}||_F^2 d_{2,T}^{2\alpha} + \bar{\gamma}_2||\boldsymbol{g}_{2,T}||_F^2 d_{1,T}^{2\alpha}\right)} \\ &= \frac{\bar{\gamma}_1 E_{g_{1,T}}[||\boldsymbol{g}_{1,T}||_F^2]\bar{\gamma}_2 E_{g_{2,T}}[||\boldsymbol{g}_{2,T}||_F^2]}{\bar{\gamma}_1 E_{g_{1,T}}[||\boldsymbol{g}_{1,T}||_F^2] d_{2,T}^{2\alpha} + \bar{\gamma}_2 E_{g_{2,T}}[||\boldsymbol{g}_{2,T}||_F^2] d_{1,T}^{2\alpha}} \\ &= \frac{\bar{\gamma}_{1,T}\bar{\gamma}_{2,T}}{\bar{\gamma}_{1,T} d_{2,T}^{2\alpha} + \bar{\gamma}_{2,T} d_{1,T}^{2\alpha}} \end{aligned} \tag{22}$$

where $\bar{\gamma}_{i,T} = \bar{\gamma}_i E_{g_{i,T}}[||\boldsymbol{g}_{i,T}||_F^2] (i = 1, 2)$. It has been proven in Ref.[17] that such kind of approximation can obtain satisfying accuracy. As such, the optimization problem with the maximal heading angle constraint can be reformulated as

$$\max_{\delta_T} \frac{\bar{\gamma}_{1,T}\bar{\gamma}_{2,T}}{\bar{\gamma}_{1,T} d_{2,T}^{2\alpha} + \bar{\gamma}_{2,T} d_{1,T}^{2\alpha}} \tag{23a}$$

$$\text{s.t.} |\delta_T - \delta_{T-1}| \leqslant \delta_{\max} \tag{23b}$$

For convenience, the above optimization problem should be equivalently expressed as

$$\min_{\delta_T} (\bar{\gamma}_{1,T} d_{2,T}^{2\alpha} + \bar{\gamma}_{2,T} d_{1,T}^{2\alpha}) \tag{24a}$$

$$\text{s.t.} |\delta_T - \delta_{T-1}| \leqslant \delta_{\max} \tag{24b}$$

Substituting Eq. (19) into Eq. (24a) yields

$$\begin{aligned} \bar{\gamma}_{1,T} d_{2,T}^{2\alpha} + \bar{\gamma}_{2,T} d_{1,T}^{2\alpha} &\approx \bar{\gamma}_{1,T} a_{2,T}^{\alpha} + \bar{\gamma}_{2,T} a_{1,T}^{\alpha} + +\bar{\gamma}_{1,T}\rho_{2,T}\cos(\delta_T \\ &\quad - \phi_{2,T}) + \bar{\gamma}_{2,T}\rho_{1,T}\cos(\delta_T - \phi_{1,T}) \\ &= \zeta_T + \psi_T\cos(\delta_T - \varphi_T) \end{aligned} \tag{25}$$

where

$$\begin{cases} \zeta_T = \bar{\gamma}_{1,T} a_{2,T}^{\alpha} + \bar{\gamma}_{2,T} a_{1,T}^{\alpha} \\ \psi_T = \left[2\bar{\gamma}_{1,T}\bar{\gamma}_{2,T}\rho_{1,T}\rho_{2,T}\cos(\phi_{2,T} - \phi_{1,T}) \right. \\ \qquad \left. + \bar{\gamma}_{1,T}^2\rho_{2,T}^2 + \bar{\gamma}_{2,T}^2\rho_{1,T}^2\right]^{\frac{1}{2}} \\ \varphi_T = \begin{cases} -\arctan\dfrac{\varsigma_T}{\tau_T} & \tau_T \geqslant 0 \\ -\arctan\dfrac{\varsigma_T}{\tau_T} + \pi & \tau_T < 0 \end{cases} \end{cases} \tag{26}$$

with

$$\begin{cases} \varsigma_T = \bar{\gamma}_{1,T}\rho_{2,T}\sin\phi_{2,T} + \bar{\gamma}_{2,T}\rho_{1,T}\sin\phi_{1,T} \\ \tau_T = \bar{\gamma}_{1,T}\rho_{2,T}\cos\phi_{2,T} + \bar{\gamma}_{2,T}\rho_{1,T}\cos\phi_{1,T} \end{cases} \tag{27}$$

As a result, the optimization problem in Eq. (24) can be rewritten as a sinusoidal function of δ_T, giving

$$\min_{\delta_T} \quad \zeta_T + \psi_T\cos(\delta_T - \varphi_T) \tag{28a}$$

$$\text{s.t.} \quad |\delta_T - \delta_{T-1}| \leqslant \delta_{\max} \tag{28b}$$

Following Ref.[11], it is not difficult to obtain the optimal heading angle as

$$\delta_T^{\text{opt}} = \begin{cases} \tilde{\delta}_T & \delta_T^{\text{l}} < \tilde{\delta}_T < \delta_T^{\text{u}} \\ \delta_T^{\text{l}} & \text{mod}_\pi|\delta_T^{\text{l}} - \tilde{\delta}_T| \leqslant \text{mod}_\pi|\delta_T^{\text{u}} - \tilde{\delta}_T| \\ \delta_T^{\text{u}} & \text{mod}_\pi|\delta_T^{\text{l}} - \tilde{\delta}_T| > \text{mod}_\pi|\delta_T^{\text{u}} - \tilde{\delta}_T| \end{cases} \tag{29}$$

where $\tilde{\delta}_T = \text{mod}_{2\pi}(\pi - \varphi_T)$, $\delta_T^l = \delta_{T-1} - \delta_{\max}$, $\delta_T^{\text{u}} = \delta_{T-1} + \delta_{\max}$, and $\text{mod}_a(b)$ denotes the modulo operation finding the remainder of division of a by b.

Assuming that the UAV can predict the AP position at time step T, we propose a simple yet effective path planning algorithm by calculating the UAV heading angle at time step $T - 1$ so that the performance of the network at time step T will be optimized. Finally, the proposed algorithm with respect to maximizing the output SNR of the considered relay network can be summarized in Fig. 2.

4. Performance analysis

Since the outage probability (OP) is an important quality-of-service (QoS) measure in wireless communication networks, we now derive the closed-form OP expression to investigate the performance of the UAV-assisted relay network conveniently. Here we use γ_T instead of $\gamma_T^{\max}$ for notional convenience.

It is well-known that the OP is defined as the probability that the SNR falls below a threshold SNR γ_{th}. The OP of the proposed relay network at time step T can be expressed as[18]

$$P_{\text{out}} = \Pr(\gamma_T \leqslant \gamma_{\text{th}}) = F_{\gamma_T}(\gamma_{\text{th}}) \tag{30}$$

where $F_{\gamma_T}(x)$ is the cumulative distribution function (CDF) of γ_T given by

01: Initialize the positions of the BS, the AP, and the UAV and the velocities of the AP and the UAV. Define $T_{\max}$ as the run time of the UAV task and set $T=1$.

02: while $T \leqslant T_{\max}$

03: $T=T+1$

04: Set the optimal BF weight vectors $\boldsymbol{w}_{i,T}^{\text{opt}}=\boldsymbol{g}_{i,T} \Big/ \left\|\boldsymbol{g}_{i,T}\right\|_F^2$ for $i=1,2$, and formulate the instantaneous output SNR expression in Eq. (16).

05: Estimate the average SNR of each link $\bar{\gamma}_{i,T}$ ($i=1,2$), and obtain the approximate average SNR in Eq. (22).

06: Obtain the AP position ($x_{1,T}$, $y_{1,T}$, 0) and the UAV position (x_{T-1}, y_{T-1}, h) at time steps T and $T-1$, respectively.

07: Calculate ζ_T, ψ_T, and φ_T in Eq. (26) and set up the optimization problem in Eq. (28).

08: Use Eq. (29) to find the solution of Eq. (28) as δ_T^{opt} for $\delta_T^{\text{opt}} \in \left[\delta_{T-1}-\delta_{\max}, \delta_{T-1}+\delta_{\max}\right]$.

09: Update $\delta_T=\delta_T^{\text{opt}}$ and calculate the UAV position (x_T, y_T, h) at time step T.

10: The UAV flies with the optimal heading angle δ_T^{opt}.

11: end while.

Fig. 2 The proposed optimal algorithm.

$$
\begin{aligned}
F_{\gamma_T}(x) &= \Pr\left(\frac{\gamma_{1,T}\gamma_{2,T}}{d_{2,T}^{2\alpha}\gamma_{1,T}+d_{1,T}^{2\alpha}\gamma_{2,T}} \leqslant x\right) \\
&= \int_0^{d_{1,T}^{2\alpha}x} \Pr\left(\gamma_{2,T} > \frac{d_{2,T}^{2\alpha}xy}{y-d_{1,T}^{2\alpha}x}\right) f_{\gamma_{1,T}}(y)\mathrm{d}y \\
&\quad + \int_{d_{1,T}^{2\alpha}x}^{\infty} \Pr\left(\gamma_{2,T} < \frac{d_{2,T}^{2\alpha}xy}{y-d_{1,T}^{2\alpha}x}\right) f_{\gamma_{1,T}}(y)\mathrm{d}y \\
&= 1-\int_{d_{1,T}^{2\alpha}x}^{\infty} f_{\gamma_{1,T}}(y)\mathrm{d}y \\
&\quad + \int_{d_{1,T}^{2\alpha}x}^{\infty} F_{\gamma_{2,T}}\left(\frac{d_{2,T}^{2\alpha}xy}{y-d_{1,T}^{2\alpha}x}\right) f_{\gamma_{1,T}}(y)\mathrm{d}y \\
&= 1-\int_{d_{1,T}^{2\alpha}x}^{\infty}\left[1-F_{\gamma_{2,T}}\left(\frac{d_{2,T}^{2\alpha}xy}{y-d_{1,T}^{2\alpha}x}\right)\right] f_{\gamma_{1,T}}(y)\mathrm{d}y
\end{aligned} \tag{31}
$$

where $f_{\gamma_{1,T}}(x)$ is the probability distribution function (PDF) of $\gamma_{1,T}$. Since both of the AP-UAV and UAV-BS links experience Rayleigh fading, $\gamma_{i,T}$ ($i=1, 2$) follows a Chi-square distribution with $2N_i$ degrees of freedom, whose PDF and CDF can be, respectively, expressed as[18]

$$
f_{\gamma_{1,T}}(x)=\frac{x^{N_1-1}}{\bar{\gamma}_1^{N_1}(N_1-1)!}\exp\left(-\frac{x}{\bar{\gamma}_1}\right) \tag{32}
$$

and

$$
F_{\gamma_{2,T}}(x)=1-\exp\left(-\frac{x}{\bar{\gamma}_2}\right)\sum_{i=0}^{N_2-1}\frac{1}{i!}\left(\frac{x}{\bar{\gamma}_2}\right)^i \tag{33}
$$

Substituting Eqs. (32) and (33) into Eq. (31) and after some mathematical computations, one can obtain

$$
\begin{aligned}
F_{\gamma_T}(x) &= 1-\sum_{i=0}^{N_2-1}\frac{1}{i!(N_1-1)!}\left(\frac{1}{\bar{\gamma}_1}\right)^{N_1}\left(\frac{d_{2,T}^{2\alpha}x}{\bar{\gamma}_2}\right)^i \times \int_{d_{1,T}^{2\alpha}x}^{\infty} \\
&\quad \times \exp\left(-\frac{d_{2,T}^{2\alpha}xy}{\bar{\gamma}_2(y-d_{1,T}^{2\alpha}x)}-\frac{y}{\bar{\gamma}_1}\right)\frac{y^{N_1+i-1}}{(y-d_{1,T}^{2\alpha}x)^i}\mathrm{d}y_{A_1}
\end{aligned} \tag{34}
$$

with the change of variables as $u=y-d_{1,T}^{2\alpha}x$, A_1 can be calculated as

$$
\begin{aligned}
A_1 &= \exp\left(-\left(\frac{d_{1,T}^{2\alpha}}{\bar{\gamma}_1}+\frac{d_{2,T}^{2\alpha}}{\bar{\gamma}_2}\right)x\right) \\
&\quad \times \int_0^{\infty}\exp\left(-\frac{1}{\bar{\gamma}_1}u-\frac{d_{1,T}^{2\alpha}d_{2,T}^{2\alpha}x^2}{\bar{\gamma}_2}\cdot\frac{1}{u}\right)u^{-i}\left(d_{1,T}^{2\alpha}x+u\right)^{N_1+i-1}\mathrm{d}u \\
&= \exp\left(-\left(\frac{d_{1,T}^{2\alpha}}{\bar{\gamma}_1}+\frac{d_{2,T}^{2\alpha}}{\bar{\gamma}_2}\right)x\right)\sum_{j=0}^{N_1+i-1}\binom{N_1+i-1}{j}(d_{1,T}^{2\alpha}x)^j \\
&\quad \times \int_0^{\infty}u^{N_1-j-1}\exp\left(-\frac{1}{-}\bar{\gamma}_1 u-\frac{d_{1,T}^{2\alpha}d_{2,T}^{2\alpha}x^2}{\bar{\gamma}_2}\frac{1}{u}\right)du
\end{aligned} \tag{35}
$$

where $\binom{N_1+i-1}{j}=\frac{j!}{(N_1+i-1)!(N_1+i-1-j)!}$ is the binomial coefficient. With the help of the following identity[19]

$$
\int_0^{\infty}x^{v-1}\exp\left(-\frac{\alpha}{x}-\beta x\right)\mathrm{d}x=2\left(\frac{\alpha}{\beta}\right)^{\frac{v}{2}}K_v\left(2\sqrt{\alpha\beta}\right) \tag{36}
$$

where $K_v(\bullet)$ is the vth-order modified Bessel function of the second kind[19], the closed-form expression of Eq. (35) is given by

$$
\begin{aligned}
A_1 &= 2x^{N_1}\exp\left(-\left(\frac{d_{1,T}^{2\alpha}}{\bar{\gamma}_1}+\frac{d_{2,T}^{2\alpha}}{\bar{\gamma}_2}\right)x\right)\sum_{j=0}^{N_1+i-1}\binom{N_1+i-1}{j} \\
&\quad \times (d_{1,T}^{2\alpha})^{\frac{N_1+j}{2}}\left(\frac{d_{2,T}^{2\alpha}\bar{\gamma}_1}{\bar{\gamma}_2}\right)^{\frac{N_1-j}{2}}K_{N_1-j}\left(2\sqrt{\frac{d_{1,T}^{2\alpha}d_{2,T}^{2\alpha}}{\bar{\gamma}_1\bar{\gamma}_2}}x\right)
\end{aligned} \tag{37}
$$

Substituting Eq. (37) into Eq. (34) yields

$$
\begin{aligned}
F_{\gamma_T}(x) &= 1-2\exp\left(-\left(\frac{d_{1,T}^{2\alpha}}{\bar{\gamma}_1}+\frac{d_{2,T}^{2\alpha}}{\bar{\gamma}_2}\right)x\right) \\
&\quad \times \sum_{i=0}^{N_2-1}\sum_{j=0}^{N_1+i-1}\binom{N_1+i-1}{j}\frac{x^{N_1+i}}{i!(N_1-1)!}\left(\frac{d_{1,T}^{2\alpha}}{\bar{\gamma}_1}\right)^{\frac{N_1+j}{2}} \\
&\quad \times \left(\frac{d_{2,T}^{2\alpha}}{\bar{\gamma}_2}\right)^{\frac{N_1+2i-j}{2}}K_{N_1-j}\left(2\sqrt{\frac{d_{1,T}^{2\alpha}d_{2,T}^{2\alpha}}{\bar{\gamma}_1\bar{\gamma}_2}}x\right)
\end{aligned} \tag{38}
$$

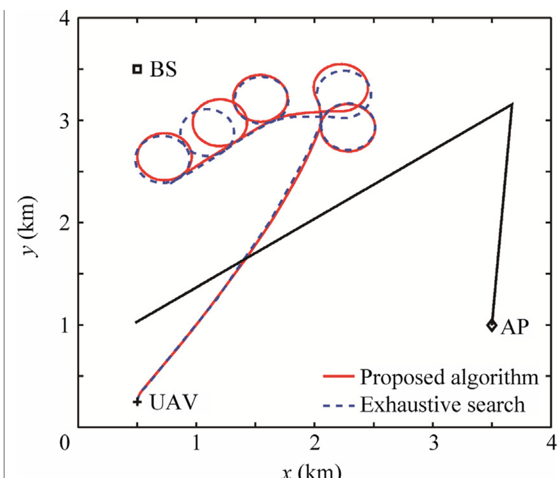
Thus, by replacing x with γ_{th} in Eq. (30), the OP of the considered relay network can be obtained directly.

5. Computer simulations

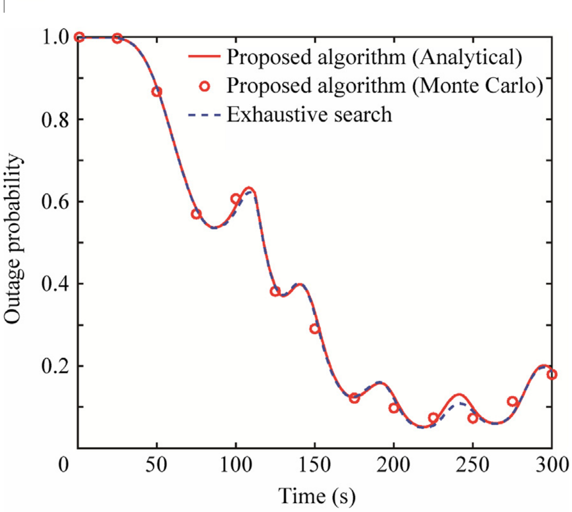
In this section, we provide computer simulations to investigate the performance of the UAV-assisted relay network with the proposed scheme. Here we consider the scenario where a mobile AP having multiple antennas communicates with a fixed BS having multiple antennas via a UAV relay with one antenna. The initial positions of the BS, the UAV, and the AP in meter are (500, 3500, 0), (500, 250, 350), (3500, 1000, 0), respectively. We assume that the UAV flies with a constant altitude $h = 350$ m and a constant velocity $v = 40$ m/s, and the AP constantly moves at 20 m/s. The time interval of updating the UAV heading angle is $\Delta T = 1$ s, and all the simulations are run for 300 time steps. At time step $T = 110$ s, the AP makes a sharp turn with $\delta_{110}/\delta_{109} = -1.7059$. In addition, the average SNRs are $\bar{\gamma}_1 = \bar{\gamma}_2 = 70$ dB, the threshold SNR for calculating the OP is $\gamma_{th} = 8$ dB, and the path loss exponent is set to be $\alpha = 1$ [11].

First of all, by assuming $N_1 = N_2 = 4$ and $\delta_{max} = 10°$, the optimal path of the UAV and the OP of the relay network versus time step T are shown in Fig. 3(a) and (b), respectively. For the purpose of comparison, we also provide the simulation results using the exhaustive search method in the same figures, which are apparently the optimal curves. As we can see, both of the flying path and the OP with the proposed algorithm are similar to those with the exhaustive search method, implying that our scheme can obtain nearly optimal performance. Since the speed of the UAV is much higher than that of the AP, the UAV is forced to fly in a tight circular path in some cases so that an optimal position to achieve the maximal SNR is maintained. In addition, from Fig. 3(b), it can also be observed that the analytical results agree well with the Monte Carlo experiments at every time step, indicating that the derived exact OP expression in Eq. (38) can accurately evaluate the performance of the considered relay network.

Secondly, Fig. 4 illustrates the impact of the antenna number on the performance of the UAV-assisted relay network. Here, we select two antenna configurations, namely, ($N_1 = 4$, $N_2 = 4$) and ($N_1 = 4$, $N_2 = 6$). It is interesting to find that when more antennas are employed at the BS, the

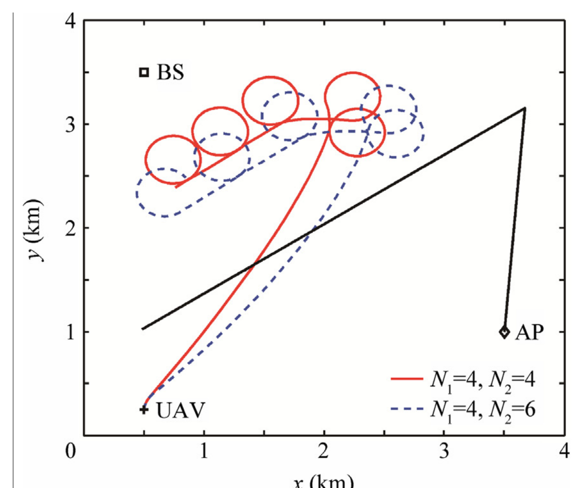


(a) Node positions and the optimal path of the UAV

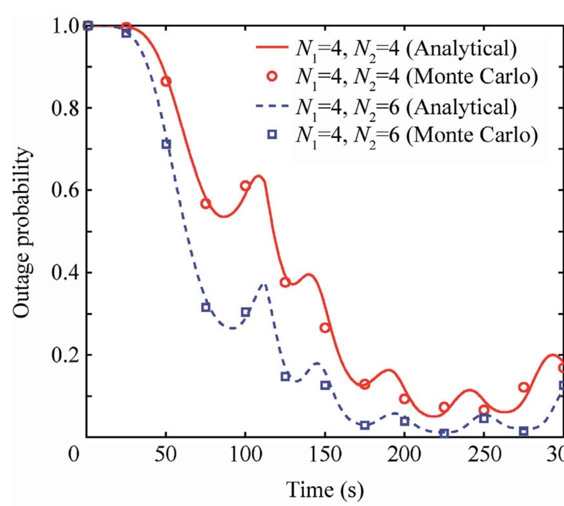


(b) The OP of the relay network

Fig. 3 Comparison of the proposed algorithm and exhaustive search with $N_1 = N_2 = 4$ and $\delta_{max} = 10°$.



(a) Node positions and the optimal path of the UAV with different antenna numbers



(b) The OP of the relay network

Fig. 4 Comparison of different antenna numbers for the relay network with $\delta_{max} = 10°$.

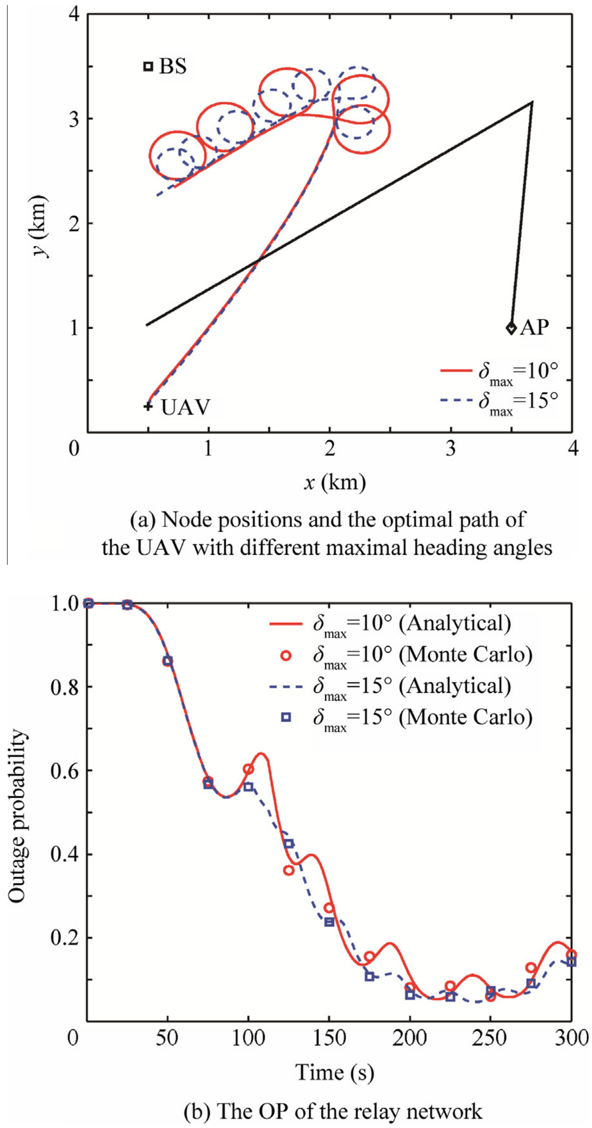


Fig. 5 Comparison of different maximal heading angles with $N_1 = N_2 = 4$.

flying path of the UAV shifts to the AP and the OP of the considered relay network is significantly reduced. This is because the increased array gain due to more antennas can compensate for the path loss of the UAV-BS link.

Finally, we select two maximal heading angles as $\delta_{max} = 10°$ and $\delta_{max} = 15°$, and provide the curves of the UAV flying path and the OP in Fig. 5(a) and (b), respectively. It can be found that the radius of the circular path and the fluctuation of the OP curve decrease with the increase of the maximal heading angle. This is because increasing δ_{max} can give the UAV more flexibility in choosing its heading angle.

6. Conclusions

(1) We have proposed an optimal design for BF and UAV path planning by obtaining the solution of the BF weight vector and the UAV heading angle. Simulation results have shown that the proposed approach can obtain nearly optimal flying path and OP performance of the considered relay network.
(2) We have derived the closed-form expression of the OP, which can accurately evaluate the performance of the relay network conveniently.
(3) The simulation results have shown that while increasing the antennas employed at the BS, the UAV flying path shifts to the AP and the OP performance of the relay network is significantly decreased.
(4) The results have also indicated that increasing the maximal heading angle of the UAV can reduce the radius of the circular path and the fluctuation of the OP performance.

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References

1. Samad T, Bay JS, Godble D. Network-centric systems for military operations in urban terrain: the role of UAVs. *Proc IEEE* 2007;**95**(1):92–107.
2. Berioli M, Nolinaro A, Morosi S, Scalise S. Aerospace communications for emergency applications. *Proc IEEE* 2011;**99**(11):1922–38.
3. Frew EW, Brown TX. Airborne communication networks for small unmanned aircraft systems. *Proc IEEE* 2008;**96**(12):2008–27.
4. Zhu H, Swindlehurst AL, Liu KJR. Optimization of MANET connectivity via smart deployment/movement of unmanned air vehicles. *IEEE Trans Veh Technol* 2009;**58**(7):3533–46.
5. De Freitas EP, Heimfarth T, Netto IF, Lino CE, Pereira CE, Ferreira AM, et al. UAV relay network to support WSN connectivity. *International congress on ultra modern telecommunications and control system*; 2010 Oct 18–20; Moscow, Russia. New Jersey: IEEE; 2010. p. 309–14.
6. Burdakov O, Doherty P, Holmberg K, Olsson P. Optimal placement of UV-based communications relay nodes. *J Global Optim* 2010;**48**(4):511–31.
7. Burdakov O, Doherty P, Holmberg K, Kvarnstrom J, Olsson P. Relay positioning for unmanned aerial vehicle surveillance. *Int J Robot Res* 2010;**29**(8):1069–87.
8. Dixon C, Frew E. Optimizing cascaded chains of unmanned aircraft acting as communication relays. *IEEE J Sel Areas Commun* 2012;**30**(5):883–98.
9. Du B, Zhang J. Relaying selection with network-coded cooperative protocols in aeronautical communications. *Chinese J Aeronaut* 2010;**23**(1):75–83.
10. Zhan P, Yu K, Swindlehurst A. Wireless relay communications with unmanned aerial vehicles: performance and optimization. *IEEE Trans Aerosp Electron Syst* 2011;**47**(3):2068–84.
11. Jiang F, Swindlehurst A. Optimization of UAV heading for the ground-to-air uplink. *IEEE J Sel Areas Commun* 2012;**30**(5):993–1005.

12. Palat RC, Annamalau A, Reed JH. Cooperative relaying for ad-hoc ground networks using swarm UAVs. *IEEE conference on military communications*; 2005 Oct 17–20; Atlantic City, USA. New York: IEEE; 2005. p. 1588–94.
13. Zhan P, Yu K, Swindlehurst L. Wireless relay communications using an unmanned aerial vehicles. *IEEE workshop on signal processing advances in wireless communications*; 2006 Jul 2–5; Cannes, France. New Jersey: IEEE; 2006. p. 1–5.
14. Rappaport TS. *Wireless Communications: Principles and Practice*. 2nd ed. New Jersey: Prentice Hall PTR; 2001.
15. Horn RA, Johnson CR. *Matrix Analysis*. 2nd ed. New York: Cambridge University Press; 2012.
16. Dubins L. On curves of minimal length with a constraint on average curvature, and with prescribed initial and terminal positions and tangents. *Am J Math* 1957;**79**(3):497–516.
17. Yi Z, Kim I. Joint optimization of relay-precoders and decoders with partial channel side information in cooperative networks. *IEEE J Sel Areas Commun* 2007;**25**(2):447–58.
18. Simon MK, Alouini MS. *Digital communication over fading channels*. 2nd ed. Hoboken: John Wiley & Sons Press; 2005.
19. Gradshteyn IS, Pyzhik IM. *Table of integrals, series, and products*. 7th ed. Burlington: Academic Press; 2007.

Ouyang Jian received his B.S. and M.S. degrees from the College of Computer Science and Technology at Nanjing University of Aeronautics and Astronautics (NUAA), Nanjing, China, in 2007 and 2010, respectively. He is currently working towards his Ph.D. degree in computer engineering at NUAA. His research interests include path planning for autonomous vehicles, cooperative relaying, and wireless relay networks.

Zhuang Yi is a professor and Ph. D. advisor at Nanjing University of Aeronautics and Astronautics (NUAA). Her research interests include wireless sensor networks and distributed computing.